Appendix for

"Boundedly Rational Search with Positive Search Costs"

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Proposition 1. There is a unique solution w* to the Kuhn-Tucker conditions that $g(w^*, V, \overline{p_c}) \leq 0$ and $w^* g(w^*, V, \overline{p_c}) = 0$.

PROOF: First note that if $g(w, V, \overline{p_c}) < 0$ for all $w \in [0, 1]$, then w* = 0 is the unique solution. Second, suppose there is some $w \in [0, 1]$ such that $g(w, V, \overline{p_c}) = 0$. It suffices to show that $\partial g(w, V, \overline{p_c})/\partial w < 0$ at such a w. Recall

$$g(w, V, \overline{p_c}) \equiv -1/(1-w) + \gamma\{1 - p_a - \beta p_a p_c/N(1-\beta\theta(w, V, \overline{p_c})]\}. \quad \text{(A1)}$$

$$\partial g(w, V, \overline{p_c})/\partial w = -1/(1-w)^2 - \gamma^2\{p_a(1-p_a) + (1-2p_a)\beta p_a p_c/N(1-\beta\theta) - [\beta p_a p_c/N(1-\beta\theta)]^2\}$$

$$= -1/(1-w)^2 - \gamma^2 p_a[1 + \beta p_c/N(1-\beta\theta)][1 - p_a - \beta p_a p_c/N(1-\beta\theta)]. \quad \text{(A2)}$$

Note that the last term in square brackets of eq(A1) is identical to the last term in curly braces { } of eq(A1). Thus, for any $w \in [0, 1]$ such that $g(w, V, \overline{p_c}) = 0$, $1 - p_a - \beta p_a p_c/N(1-\beta\theta(w)] = 1/\gamma(1-w) > 0$; hence, by eq(A2), $\partial g(w, V, \overline{p_c})/\partial w \leq -1/(1-w)^2 \leq -1$. Q.E.D.

Proposition 2. There is a unique solution w* to the Kuhn-Tucker conditions that $G(w^*, V) \leq 0$ and $w^* G(w, V) = 0$.

PROOF: $G(w, V) \equiv g[w, V, p_c(w, V)] = -1/(1-w) + \gamma[1 - p_a - \beta p_a p_c/N(1-\beta p_c)]$. It will suffice to show that $\partial G(w, V)/\partial w < 0$ for all $w \in [0, 1]$.

$$\partial G(w, V)/\partial w = -1/(1-w)^2 - \gamma^2\{p_a(1-p_a) + (1-2p_a)\beta p_a p_c/N(1-\beta p_c) - N(\beta p_a p_c/N(1-\beta p_c))^2\}$$

$$= -1/(1-w)^2 - \gamma^2 p_a\{[1 - p_a - \beta p_a p_c/N(1-\beta p_c)] + (\beta p_c/N(1-\beta p_c))[1 - p_a - \beta p_a p_c/(1-\beta p_c)]\}. \quad \text{(A3)}$$

Observe that since $N \geq 1$, $1 - p_a - \beta p_a p_c/N(1-\beta p_c) \geq 1 - p_a - \beta p_a p_c/(1-\beta p_c)$, and since $1 - p_a > p_c$, the last term is greater than or equal to $p_c[1 - \beta p_a/(1-\beta p_c)] \geq 0$. Hence, both terms in curly brackets { } of eq(A3) are non-negative, implying that $\partial G(w, V)/\partial w \leq -1/(1-w)^2 \leq -1$. Q.E.D.