Boundedly-Rational *vs.* Optimization-Based Behavior: A Distinction without a Difference

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ABSTRACT

We prove that every well-defined behavioral rule can be interpreted as being optimization-based. Therefore, the categorical distinction between boundedly-rational behavior and optimization-based behavior asserted by Harstad and Selten (2013) does not entail a substantive difference.

1. Introduction.

While we all recognize that humans are fallible, perhaps the primary reasons most economists have been reluctant to pursue models of bounded rationality are (i) the lack of a formal definition of bounded rationality, and (ii) the belief that perfect rationality is a reasonable approximation to whatever is meant by bounded rationality.

In their essay delineating the challenge facing boundedly-rational models of behavior in competition with neoclassic economic models, Harstad and Selten (2013) make a distinction between boundedly-rational behavior and optimization-based behavior, but do not provide formal definitions. Nonetheless, one surmises that Harstad and Selten include "noisy optimization" within their concept of optimization-based behavior, and that in their opinion truly boundedly-rational behavior is not optimization-based. Crawford (2013, p513) states: "Optimization-based models maintain the customary neoclassical assumption that individuals act as if to optimize *something*." While not questioning Harstad and Selten's distinction, Crawford argues that even Selten's "directional learning" and "impulse equilibrium" models can be considered optimization-based. Both Crawford (2013) and Rabin (2013) stress the merits of optimization-based models of behavior.

In this essay it is argued that the categorical distinction between boundedly-rational behavior and optimization-based behavior does not entail a substantive difference. One obvious argument is that "boundedly-rational behavior" is necessarily optimization-based by virtue of being a modification of "rational behavior" which entails maximization of an objective function. We take the approach of first providing a general mathematical definition of *behavior* (without any prefix), and then showing that every such behavior can be interpreted as being optimization-based. Implications of this result are discussed in the last section.

2. The Formal Argument.

According to the *Oxford Dictionary of English*, ***behavior*** is "the way in which an animal or person acts in response to a particular situation or stimulus." To be mathematically precise, the pertinent domain of behavior entails a Decision Maker (DM) who, at time t, faces a countable set of feasible actions A_t, and has other information x_t. The pair (A_t, x_t) is a "situation". *Behavior* for the DM is a function that maps situations into probability measures on the feasible actions, $\Delta(A_t)$. For some situations, the behavior may put probability one on a unique action; otherwise it at least specifies the probability of each feasible action.

To distinguish this mathematical definition of behavior from alternative definitions, we will call the former a *behavioral rule*, and denote it by ρ.[1] Given situation (A_t, x_t), the function $\rho(A_t, x_t)$ is a probability distribution on A_t, and $\rho(a \mid A_t, x_t)$ denotes the probability that action $a \in A_t$ will be the outcome. Observe that $\rho(A_t, x_t)$ could be nearly uniform on A_t, and at the other extreme it could put probability 0.999 on the action that maximizes some objective function, in which case we can say it is optimization-based.

Let $u(a \mid A_t, x_t)$ be any positive affine transformation of $ln[\rho(a \mid A_t, x_t)]$:

$$u(a \mid A_t, x_t) = \sigma\, ln[\rho(a \mid A_t, x_t)] + c, \tag{1}$$

where $\sigma > 0$. Then, we can express the behavioral rule in terms of u( ) as follows:

[1] This definition is an extension of Stahl (1999), although there may be earlier precedents.

$$\rho(a \mid A_t, x_t) \;=\; \frac{exp[u(a \mid A_t, x_t)/\sigma]}{\sum_{a' \in A_t} exp[u(a' \mid A_t, x_t)/\sigma]}\,.^{2} \qquad (2)$$

The right-hand side of eq(2) is the familiar logistic function, but it arises directly from the specification of the behavioral rule and not from any structural model of the decision process. On the other hand, we are free to interpret eq(2) as the probabilistic choice function of a DM with objective function $u(a \mid A_t, x_t)$ and extreme-valued additive noise with variance σ^2. Since this interpretation is optimization-based, clearly every behavioral rule can be interpreted as being optimization-based. Q.E.D.

Since syntactically "optimization-based behavior" is a subset of behavior, it follows that **the class of behavioral rules and the class of optimization-based behaviors are identical**. Therefore, the class of non-optimization-based behaviors is disjoint with the class of behavioral rules. But then, if we adopt Harstad and Selten's view that truly boundedly-rational behavior is not optimization-based, we are forced to conclude that the class of truly boundedly-rational behaviors is empty. Alternatively, we can adopt the more common view that boundedly-rational behavior means any behavior except exact optimization.[3]

While exact optimization could be included in the class of "optimization-based" decision processes, it is fair to infer that Harstad and Selten intended the term "optimization-based" to exclude exact optimization. Under this interpretation and the common view of bounded rationality, the categorical distinction between boundedly-rational behavior and optimization-based behavior does not entail a difference, since the classes of behavior coincide.

3. Implications.

Of course, the mere existence of a mathematical isomorphism between two formalisms does not provide a justification for favoring the associated interpretation of one over the other. Economists are quite familiar with situations in which a constrained maximization formalism is more convenient for some questions, and the dual constrained minimization formalism is more convenient for other questions. Similarly, if we desire an estimate of a behavioral rule, then we

[2] We adopt the conventions that *ln*(0) = $-\infty$, and *exp*($-\infty$) = 0. Note that since $\sum_{a \in A_t} \rho(a \mid A_t, x_t) = 1$, the denominator of eq(2) is always strictly positive. When A_t is not countable, $\rho(a \mid A_t, \omega_t)$ is a probability density, and the summation in the denominator of eq(2) is replaced by an integral.

[3] It is well-known that exact maximization of a utility function can result in multiple optimal actions with no implication regarding the probability of any particular optimal action. Therefore, additional assumptions are needed before the notion of perfect-rationality implies a well-defined behavioral rule.

must impose the constraint that $\rho(A_t, x_t)$ lies in the simplex $\Delta(A_t)$. In contrast, $u(\bullet \mid A_t, x_t)$ is unconstrained on the real line, so estimating $u(\bullet \mid A_t, x_t)$ and using eq(2) is computationally easier. Nevertheless, this computation-based advantage does not imply that the optimization-based interpretation should be favored over the rule-based interpretation.

Harstad and Selten (2013) suggest six advantages neoclassical microeconomics has over alternative models: (1) the range of application, (2) coherence, internal structure and teachability, (3) isolation of economic forces and definitiveness, (4) focusing empirical studies, (5) solution concepts and stationarity, and (6) efficiency conclusions and minimal value judgments. Advantages 1-4 arise from the hypothesis of a utility function, so by virtue of the isomorphism given by eq(1), these advantages apply to rule-based alternatives as well, and hence they are not valid reasons to favor the optimization-based interpretation. Regarding advantage 5, one could argue that the stationary equilibrium solution concept is at best an empirically empty tautology and at worst an empirically irrelevant limiting case. Further, regarding advantage 6, that the value judgments implicit in welfare analysis are minimal is itself a value judgment not without considerable controversy.[4]

One criteria for having a favorite interpretation is the ease to which it can be placed within an broader theory. For example, the behavioral rule interpretation can easily be placed within an evolutionary model. In contrast, the optimization-based interpretation needs additional assumptions concerning the nature of the DM and the decision process. For example, it is common to assume that the decision process is an exercise of the conscious freewill of the DM, and hence a stance must be taken on the mind-body problem. However, there is no conclusive scientific evidence or logical argument that consciousness is a necessary component of the decision process. Further, freewill is a metaphysical concept that should not be an integral part of any truly scientific theory.

Perhaps the most common and seemingly persuasive advantage of neoclassical microeconomics (and hence the optimization-based interpretation) is that it allows us to address and answer *welfare* questions. Does a policy make a DM better off or worse off? Is an allocation of goods Pareto optimal?

Within the rule-based interpretation, since by eq(1), $u(\bullet)$ is merely an affine transformation of the behavioral rule, its interpretation as a utility function is not valid. Therefore, the concept of Pareto optimality (defined in terms of utility) cannot be applied.

[4] For example, it is not widely appreciated that the use of the concept of aggregate consumer surplus implicitly assumes that the weight given to each individual is inversely proportional to that individual's marginal utility of income, and hence (assuming utility is concave in income) positively related to that individual's income.

On the other hand, one could attempt to recast Pareto optimality in terms of voting behavior. Consider a binary ballot between option a^0 and a^1, in which a vote *for* an option is cast iff the DM strictly prefers that option to the other, and the DM abstains if indifferent. Then, one could declare a^0 *Pareto optimal* if it receives some *for* votes against every feasible alternative a^1. This voting-based definition is equivalent to the standard utility-based definition <u>if and only if</u> each DM's voting behavior is degenerate: i.e. the probability measure on the three possible votes {for a^0, for a^1, abstention} puts probability 1 on one and only one option. In contrast, if any DM has non-degenerate voting behavior, then no a^0 can be Pareto optimal, since for any alternative a^1 there is a positive probability that a^0 could receive zero *for* votes. In other words, Pareto optimality becomes an empty concept.[5]

Nonetheless, policy analysis can still be conducted based on behavior. We can ask how likely is it that option a^1 would obtain more votes in a binary ballot with the status quo a^0, and use our best estimates of the behavioral rules to predict the outcome. A community of DMs could adopt a constitution that declares a^1 the winner iff a^1 gets more that 50% (or 66.6%, etc.) of the vote. Empirically the population distribution of behavior could have a median voter which would guarantee acyclicity of the resultant ordering.

Moreover one could still define a DM's willingness-to-pay (WTP) by hypothetically pitting a continuum of alternatives $a^1(\tau)$ against a^0, where τ is the amount of tax the DM would pay in the event alternative a^1 were adopted. In standard neoclassical economics, we would define DM i's WTP as that tax τ_i such that $u_i[a^1(\tau_i) \mid x_t] = u_i(a^0 \mid x_t)$. Under the behavioral rule interpretation, the condition on utilities would be replaced by the condition that the probability of voting for $a^1(\tau_i)$ over a^0 equals ½. These WTPs could then be aggregated and compared to costs, just as we do in standard neoclassical economics. Hence, it is not clear that there would be any practical loss from dispensing with the optimization-based interpretation.

Finally, applying this essay's critique to the literature on the evolution of preferences[6], we argue that it would be at least as useful to focus on the evolution of behavioral rules and dispense with the intermediate representation of preferences.

[5] Note that this conclusion also applies to neoclassical economic theory as soon as we allow non-degenerate probabilistic choice (e.g. standard discrete choice models). In these cases, the choices are probabilistic because of individual errors or because we (the outside observer) cannot observe the true utility function without error.
[6] See Robson and Samuelson (2010) and the references therein.

References

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